Application of the Teager Kaiser Energy Operator to Machine Diagnostics

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Abstract

The Teager Kaiser Energy Operator (TKEO) was originally developed for tracking the "energy" in speech signals, including both potential and kinetic energy, where for real-time it can be calculated very efficiently from three adjacent time samples. It has recently been shown that the TKEO is equal to the squared envelope of the derivative of the measured parameter, so the squared envelope of both the original signal and its derivative are most efficiently obtained by Hilbert transform operations via the frequency domain, including zero phase shift bandpass filtration to extract the modulated mono-carrier, and differentiation of the bandpass filtered signal by multiplication by $j\omega$ over the selected band. The paper shows the advantages of the frequency domain methods of obtaining the TKEO for machine diagnostics, including determination of the instantaneous speed of a machine.

Keywords: Teager Kaiser energy operator, frequency domain Hilbert transform techniques, machine diagnostics, amplitude and frequency demodulation, machine speed determination.

Introduction

In [1], Kaiser formalised an "energy operator" first proposed by Teager for use in speech analysis, as well as representing it as analogous to the total energy of a spring/mass system, both kinetic and potential, which continuously alternate in an oscillatory system. It has become known as the Teager Kaiser Energy Operator (TKEO). It was shown that in a discretised version it could be very efficiently estimated from three adjacent samples, effectively in real-time, as appropriate to speech analysis. In [2, 3] Maragos et al. made extensive further developments, including the use of the TKEO for amplitude and phase demodulation, and presenting error estimates for both the continuous and discretised versions.
A number of authors have proposed using the TKEO for machine diagnostics [eg 4, 5, 6], some even claiming that it gave better results for amplitude and frequency demodulation than Hilbert transform techniques. The aim of this paper is to show that where estimations do not have to be made in real-time, the most efficient (and accurate) way to estimate the TKEO is by using Hilbert transform techniques via the frequency domain, as it allows non-causal, zero phase shift, signal processing operations to be used. Machine diagnostics is a typical case where real-time operation is not required, as information is usually being sought days, weeks or months in advance of when the condition may become serious, and certainly the few seconds' delay involved in non-real-time processing is immaterial. The disadvantage of using non-causal post processing FFT techniques is the wraparound errors associated with the circularity of the FFT algorithm, but transform sizes can be made very large, and the small affected sections at the ends can usually be discarded.

Formulations and Equations

The TKEO is defined in both continuous and discretised forms, as given in Equations (1) and (2), respectively.

$$\Psi_c\left(x(t)\right)=\left[\dot{x}(t)\right]^2-x(t)\ddot{x}(t) \tag{1}$$

$$\Psi_d\left(x(n)\right)=\left[x(n)\right]^2-x(n+1)x(n-1) \tag{2}$$

where $x(t)$ is an amplitude and frequency modulated mono-component whose amplitude $A(t)$ and frequency $\omega(t)$ are slowly varying.

Taking up Kaiser's analogy of $x(t)$ as the response of a spring/mass system, the kinetic energy (KE) of the mass is proportional to the square of the velocity (the first term on the right of (1), while the potential (strain) energy (PE) in the spring is proportional to the second term on the right of (1), and these add to the total energy (KE + PE) in the system, which is given by the TKEO in (1). In [7] it is shown that the square roots of the two energy terms are Hilbert transforms of each other, and in quadrature, so that the total energy at any time is the sum of the squares and thus equal to the squared envelope of either, but in particular the squared envelope of $\dot{x}(t)$.

Thus:

$$\Psi_c\left(x(t)\right)=\mathrm{Envsq}\left(\dot{x}\left(t\right)\right) \tag{3}$$

In [2] it is shown that when the rates of change of $A(t)$ and $\omega(t)$ are within specified limits, the TKEO can be written in terms of them as:

$$\Psi_c\left[x(t)\right]\approx\left[\omega(t)\right]^2\left[A(t)\right]^2 \tag{4}$$

and the errors involved in this approximation are detailed in [2].

Since $\left[A(t)\right]^2=\mathrm{Envsq}\left(x\left(t\right)\right)$, it follows from (3) and (4) that:

$$\left[\omega(t)\right]^2=\frac{\mathrm{Envsq}\left(\dot{x}\left(t\right)\right)}{\mathrm{Envsq}\left(x\left(t\right)\right)} \tag{5}$$

Comparison of Time domain and Frequency Domain Methods

In [2] formulas are developed for achieving amplitude and frequency demodulation, using the TKEO. The formula for frequency demodulation, for example, is similar to Eq. (5), but uses the TKEO and its derivative instead of the signal and its derivative. In [4], the TKEO methods are compared with Hilbert transform methods for a number of cases, and the TKEO method shown to be better for determining the instantaneous frequency of a chirp signal with constant amplitude but frequency varying linearly between zero and 300 Hz. However, in [7] this comparison is taken up in detail, and it is shown that the difference is mainly between time domain and frequency domain execution. When the Hilbert transform is obtained by time domain convolution, the result is almost as good as the TKEO method (although the fact that only three adjacent samples are used for the latter does give smaller end effects). It is also shown that the circularity of the FFT operation is the primary reason for the very large end effects, as an abrupt change from zero to 300 Hz gives a large discontinuity. However, [7] points out that in machine diagnostics such a large frequency sweep is virtually never encountered, as most machine signals, such as from gears, contain multiple harmonics of important forcing frequencies. The maximum frequency sweep which can then be demodulated is 2:1 before the sidebands around the second harmonic overlap with those around the first harmonic. In other words, the signal would always have to be band-pass filtered to extract a single carrier mono-component. If the actual frequency sweep range is greater than 2:1 the signal must be divided into shorter overlapping sections, in each of which the frequency range is allowable (max. 2:1, but possibly less, as described in [8]). Figure 1, from [7], compares the results of using frequency domain Hilbert transform demodulation on the full range chirp with one having frequency range limited to 160-300 Hz. It is seen that the end effects are much smaller.

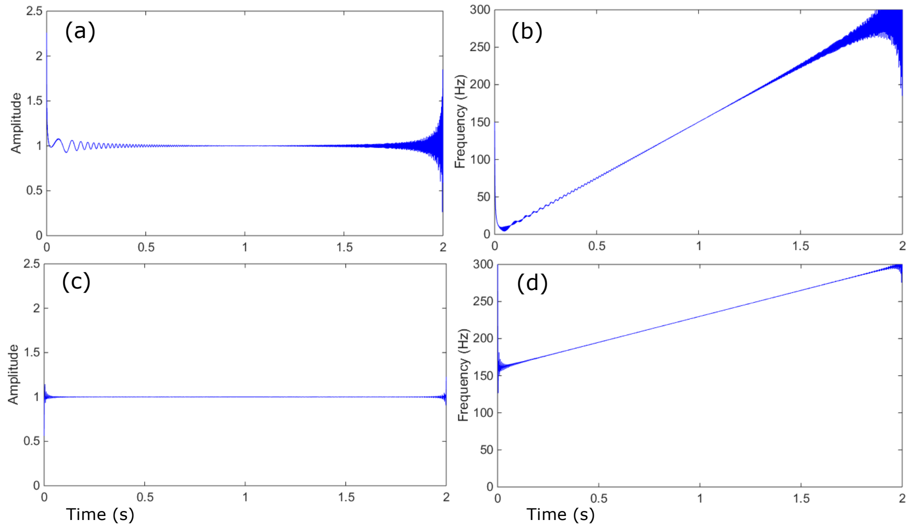

Fig. 1: Results for the two chirps using TKEO via HT in frequency domain (a, b) 0-300 Hz (c, d) 160-300 Hz (a, c) Amplitude (b, d) Frequency

As a matter of interest, the results of the two methods applied to the actual data in Ref. [4] were virtually identical, as the machine speed was constant.

The fact that a bandpass filter must be applied in general to isolate a mono-component carrier leads to a genuine difference between the real-time time domain TKEO method and the Hilbert transform approach applied in the frequency domain, as bandpass filtering in the frequency domain can be done with a non-causal, zero phase shift, ideal filter, simply by setting frequency lines outside the band to zero. It should be noted that such FFT operations are non-causal, since the second half of each time record represents negative time, in the same way that the second half of the spectrum represents negative frequency. Causal real-time filters give phase distortion and have a far from ideal filter characteristic. Frequency domain implementation of the Hilbert transform simply means inverse transforming a one-sided spectrum (positive frequencies only), this giving an analytic signal whose imaginary part is the Hilbert transform of the real part, and there is no phase distortion within the retained band. The squared envelope of the time signal is simply the squared amplitude of the analytic signal. The spectrum of the derivative of the signal, bandpass filtered in the same band, is obtained simply by multiplying the (complex one-sided) spectrum by $j\omega$, once again with no phase distortion (as there would be with a real-time differentiator). The evaluation of instantaneous frequency by Eq. (5), for example, can thus be done very efficiently via Hilbert transform techniques in the frequency domain by processing the same spectral band with and without multiplication by $j\omega$.

It thus appears that there are considerable benefits of the frequency domain approach for machine diagnostics, where real-time operation is not necessary, the only disadvantage being the wraparound effects at the ends of time records because of the circularity of the FFT transform. These can usually be truncated if a slightly longer record can be processed, as illustrated by the application examples of the next section, and the extreme example of Fig. 1(c, d).

Machine Diagnostic Applications

It has already been mentioned that there was no appreciable advantage of the TKEO vs Hilbert based demodulation in the application to a constant speed wind turbine gearbox in [4], but the TKEO did appear to give some benefit in [6], by being more sensitive to increasing impulsiveness of gear fault signals. However, the greater weighting given to higher harmonics by the differentiation associated with the TKEO, which brought this about, was only possible by ignoring the mono-component requirement of the TKEO and taking account only of the squared envelope of the signal over a much greater frequency range.

In [5], the claim was made that bearing diagnostics was improved by both the squared amplitude and squared frequency terms in the TKEO, but the latter was based on a faulty simulation model of bearing signals, assuming them to be perfectly periodic and with continuous unwrappable phase, something which does not apply in practice. This was discussed in detail in [7]. In the application to a real signal in [5], it is pointed out that the spectrum of the TKEO for a particular fault shows one more harmonic than the equivalent "envelope spectrum", but this appears to be solely due to the fact that the TKEO is a squared envelope, and thus has twice the bandwidth for exponentially damped impulse responses. The advantages of analysing the squared envelope were first made clear in [9] in the year 2000.

In [7] a comparison is made of bearing diagnostics (at constant speed) using a number of different methods, but in all cases restricting the analysed band to the range 8-24 kHz. This was basically to eliminate interference from gear signals, all located below 8 kHz, but since the frequency range is 3:1, the analysed band is obviously not a mono-component. Figure 2, from [7], compares the signals and spectra of the envelope of the acceleration (a, e), the squared envelope of the acceleration (b, f), the squared envelope of the jerk (derivative of the acceleration), (c, g), and the TKEO (d, h), theoretically the same as the squared envelope of the jerk, but evaluated by the time domain formula. The spectrum of the envelope of the

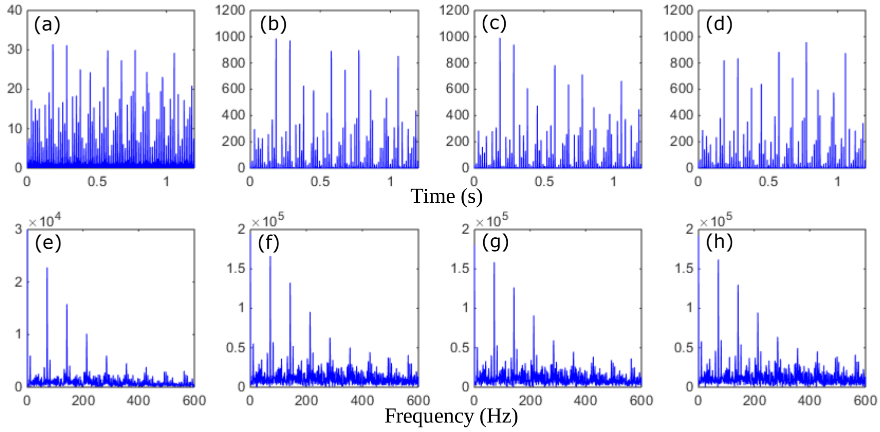

Fig. 2: Comparison of various envelope signals and their spectra
(a) Envelope of acceleration (b) Squared envelope of acceleration
(c) Squared envelope of jerk (d) TKEO (e-h) Corresponding spectra

acceleration, Fig. 2(e), does have smaller harmonics than the other three (squared envelope) spectra, but the latter are all very similar, showing that in this particular case, the differentiation to jerk has not had a large effect on the impulsiveness of the fault responses. Even though the total frequency range is 3:1, it is seen in Fig. 9 of [7] that the peak of the demodulated band has only been shifted by a few percent by the differentiation.

A more promising application of the frequency domain TKEO in machine diagnostics is its use for determining the instantaneous speed of a machine, as investigated in [10]. Figure 3 shows a result from [10] where Eq. (5) has been used on two signals to determine the speed of a gearbox. One signal is from a 2-pulse-per-rev tacho signal on the output shaft of the gearbox and the other from the first harmonic of the acceleration signal of the input shaft. Fig. 3(a) compares the smoothed results. There is no absolute measure of the correct result, though the tacho could be expected to be more accurate. However, outside the end effect zones, the maximum difference is 0.24% and standard deviation 0.03%.

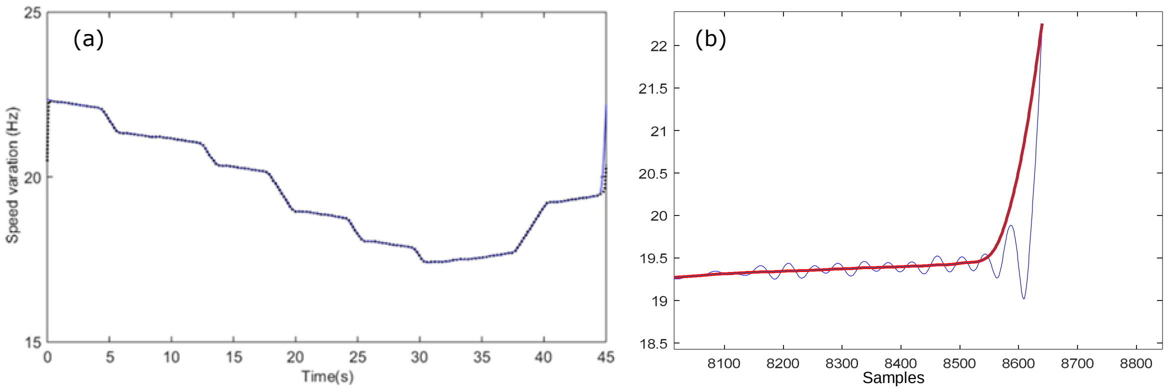

Fig. 3: (a) Comparison of speed estimates from acceleration (blue, solid) and tacho (black, dotted) adjusted for ratio (b) Zoom on end effects for acceleration signal wraparound error (blue) smoothed result (black).

Figure 3(b) is a zoom on one end of the unsmoothed and smoothed result for the acceleration signal. As described in [10] the smoothing was done by a zero phase shift moving average filter of length 100 samples. It is seen that the extent of the effects of both the wraparound error and the smoothing filter is of this order, and could be removed by truncation.

Conclusion

This paper discusses the application of the Teager Kaiser Energy Operator (TKEO) to machine diagnostic problems, and finds that many previous claims of advantages over Hilbert transform methods of demodulation are unjustified. This is largely because real-time operation, an advantage with respect to the original application of speech analysis, is not relevant in the case of machine diagnostics, so non-causal postprocessing techniques can be used. It is demonstrated that the TKEO is nothing other than the squared envelope of the derivative of a signal, so can in fact be implemented using Hilbert transform techniques via the frequency domain. Ideal bandpass filtration and differentiation can then be performed with zero phase error, giving advantages over the time domain approach. Bandpass filtration would normally be required to isolate a single mono-component signal, with a maximum frequency range of 2:1, but then the potential advantage of the differentiation, enhancement of high frequency components, is largely lost. There is a strong argument for relinquishing this requirement for gear and bearing diagnostics, and using only the squared amplitude component of the TKEO, but the question can then be asked "why not perform multiple differentiations?".

The frequency demodulation property is shown to be an efficient way of determining the instantaneous speed of a machine. The only disadvantage of frequency domain implementation, wraparound effects at the ends of the record, can usually be removed by truncation.

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