Separation of mechanical source vibrations under variable speed conditions

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Abstract

Mechanical signals are often a mixture of numerous components generated by distinct sources. The separation of these components is of high interest for machine condition monitoring, diagnosis and prognosis. An efficient way to accomplish this task is to apply a linear periodically time-variant filter, also known as the cyclic Wiener filter (CWF), assuming the signal to be cyclostationary. This requires a quasi-constant operating speed which can be very restrictive in practice. This paper addresses this issue by proposing an extension of the CWF to the variable speed case, exploiting the angle-time cyclostationary property of these signals. It is shown that the resulting filter has linear periodically angle-varying structure whose coefficients can be easily derived. The effectiveness of this filter as well as its superiority to the CWF is demonstrated on real vibration signals where the aim is to extract a bearing fault component from its noisy environment.

Keywords: Blind signal separation, cyclostationary, variable speed, cyclic Wiener filter, linear periodically angle-varying filter, angle-time cyclostationary.

Introduction

Mechanical signals are often a mixture of numerous components generated by distinct sources. The separation of these components is crucial for machine signal processing and can help in localizing, identifying and evaluating defects in mechanical systems. Cyclostationarity is a powerful framework to describe and process rotating machine signals. Under this assumption, the best way to achieve signal separation is to apply a linear periodically time-variant filter, also known as the cyclic Wiener filter (CWF), being the optimal solution in the least mean square sense [1]. Though an a priori knowledge of the signal periodicity is required, this filtering can still be referred as "blind signal separation" since no reference signal is used for the extraction of the signal of interest. Whereas the theory of the CWF was established by W. Gardner in 1993 [1], this filter was firstly applied in mechanical applications by Bonnardot et al. 2005 [2]. As advocated in Ref. [3], the capacity of the CS framework in describing machine signals is confined to the stationary regime case— i.e. when the operating speed is constant or stationary. Otherwise, cyclostationarity is jeopardized and the CWF turns unreliable no matter the signal is processed in time or angle. In details, rotating machines witness a repetitive occurrence of short-time events being related to its regular operation (such as combustion, piston slap, etc.) or to a certain dysfunction (e.g. a local fault). These events are likely to produce transient signatures whose properties are related to the system dynamics. These transients are time-invariant as they are typically dictated by time-differential equation. However, the recurrence of the events (and the transients) is due to the rotating motions of machine components. It is thus dependent on the machine kinematics and its evolution is inherently locked with the machine rotational angle. It is the presence of this (angle\time) duality what makes rotating machine signals unsuitable to be analysed within the cyclostationary (CS) framework when the systems operate under variable speed conditions. Consequently, the efficiency of the CWF is compromised since its periodicity can no longer capture the time-varying periods of speed-varying signals. Conversely, proceeding with the same approach in the angular domain (e.g. after order tracking the temporal signal) would accommodate with the signal periodicity, but will certainly misestimate its time-dependent coefficients. This issue has been recognized in previous works and some solutions have been provided. Mostly, solutions are based on windowing the transients through a finite energy window whose bandwidth is constant in time and whose position is dictated by the angle through an explicit dependence. As far as the authors know, the formalization of such a filter has not been addressed yet. This paper comes in this context aiming at filling in this gap by extending the CWF to the variable speed case, exploiting the more general framework called "angle-time cyclostationarity". This framework was specifically designed in Ref. [4] to describe rotating machine signals recorded under nonstationary operating speed. The paper is organized as follows: section 2 briefly reviews angle-time cyclostationarity and formulate the problem of blind signal separation. Section 3 proposes a solution based on the extension of the CWF to the variable speed case. Section 4 demonstrates the effectiveness of this approach on real vibration signal measured from the high-speed gearbox of a wind turbine. The paper is sealed with a conclusion in section 5.

Problem formulation

This section aims at formalizing the problem of source separation of mechanical sources through a scientific framework. As stated in the introduction, rotating machine signals recorded under speed-varying conditions are assumed angle-time cyclostationary, so the properties of this class of signals is first reviewed. Then, the blind signal separation problem is formalized.

Angle-time cyclostationarity

A signal $s(t)$ is said to exhibit angle-time cyclostationarity with cyclic orders (with unit "event per revolution") $\alpha_i \in \mathcal{A}$, if its angle-time autocorrelation function,

$$\mathfrak{R}_{2s}(\theta,\tau) \quad = \quad \mathbb{E}\left\{s\big(t(\theta)\big)s\big(t(\theta)-\tau\big)^*\right\}. \qquad (1)$$

is periodic with respect to θ (θ and t respectively denote the angle and time variables) [4]. Note that it has been assumed that a full cycle corresponds to 2π radians, that is $\theta(t)/2\pi$ expresses the evolution of the angle in terms of cycles (or revolutions).The latter condition implies the presence of non-zero Fourier coefficients associated with the *order spectrum* $\alpha_i \in \mathcal{A}$, viz

$$\mathfrak{R}_{2s}^{\alpha_i}(\tau) \quad = \quad \lim_{\Phi\to\infty}\frac{1}{\Phi}\int_{-\infty}^{+\infty}\mathfrak{R}_{2s}(\theta,\tau)\mathrm{e}^{-\mathrm{j}\alpha_i\theta}\mathrm{d}\theta \qquad (2)$$

Its Fourier transform

$$S_{2s}^{\alpha_i}(f) \quad = \quad \int_{-\infty}^{+\infty}\mathfrak{R}_{2s}^{\alpha_i}(\tau)\mathrm{e}^{-\mathrm{j}2\pi\tau f}\mathrm{d}\tau \qquad (3)$$

defines the *order-frequency cyclic power spectrum* (OFCPS) at α_i having as unit [U^2/Hz] ([U] is the signal unit), where f stands here for the *spectral frequency*. The OFCPS can be equivalently expressed in a more compact form as [4]:

$$S_{2s}^{\alpha_i}(f) \quad = \quad S_{\check{s}_\alpha \check{s}_0}(f) \qquad (4)$$

where $S_{xy}(f) \stackrel{\text{def}}{=} \mathbb{E}\{dY(f)^* dX(f+\alpha_i)\}/df$ is the cross-power spectrum and

$$\begin{aligned} \tilde{s}_\alpha(t) &= s(t)\exp\big(j\alpha\theta(t)\big)\sqrt{\omega(t)/\overline{\omega}} \\ &= s_\alpha(t)\sqrt{\omega(t)/\overline{\omega}} \end{aligned} \tag{5}$$

where ω stands for the angular speed in [rad/s], $\overline{\omega} = \lim_{T\to\infty}\left(\int_0^T \omega(t)dt\right)/T$ and $s_\alpha(t) = s(t)\exp\big(j\alpha\theta(t)\big)$.

Blind signal separation

The *blind signal separation* addressed in this paper specifically concerns the recovery of a *signal of interest*, being of AT-CS nature from a measurement which includes the latter with an unknown number of interferences and noises. Denoting by $s(t)$ the AT-CS signal of interest with cyclic orders $\alpha_i \in \mathcal{A}$, the "noisy measurement" can be expressed as follow:

$$x(t) = s(t) + n(t) \tag{6}$$

where $n(t)$ stands for the noise being nonstationary in general and potentially exhibiting angle-time cyclostationarity with a cyclic order set $\mathcal{B}$. The issue of blind signal separation turns to find an optimal estimate of the signal of interest, say $\hat{s}(t)$, by filtering $x(t)$ through a filter $h(t,u)$ (generally linear time-variant)

$$\hat{s}(t) = \int_{-\infty}^{+\infty} h(t,u)\mathrm{x}(u)\mathrm{d}u \tag{7}$$

such as to minimize the mean square error between the estimate and the signal of interest, i.e.

$$h(t,u) = argmin\{\|s(t) - \hat{s}(t)\|_2\}. \tag{8}$$

In the stationary case, the solution to the problem above is a linear-time-invariant filter (i.e. $h(t,u) = h(u)$) known as the Wiener filter, whereas the solution is linear periodically time-varying filter (also coined as the cyclic Wiener filter) in the CS case.

Proposed solution

The extraction of speed-varying (i.e. AT-CS) signals is fundamentally different from the constant-speed (i.e. CS) case as the angle-time relation turns non-linear. This subsection aims at filling in this gap by changing the structure of such filter and extending its calculation to the more general AT-CS case.

Linear periodically angle-varying (LPAV) filters

For the filter to be able to optimally detect an AT-CS signal, its structure must jointly consider the angle and the time variables. Such a filter must then have a *linear periodically angle-varying* (LPAV) structure. For an angular periodicity defined by the cyclic orders $\alpha_k \in \mathcal{H}$, such a filter takes the following form

$$\begin{aligned} h(t,u) &= \sum_{k=1}^{L} h_k(t-u)e^{-j\alpha_k\int_0^u \omega(l)dl} \\ &= \sum_{k=1}^{L} h_k(t-u)e^{-j\alpha_k\theta(u)} \end{aligned} \tag{9}$$

where $h_k(t)$ are also LTI filters and $L = card\{\mathcal{H}\}$ denotes the cardinality of the cyclic order set. It can be shown that the output-input relation of such a filter writes as

$$\begin{aligned}\xi(t) &= \int_{-\infty}^{+\infty} h(t,u)\varepsilon(u)\mathrm{d}u \\ &= \sum_{\alpha_k \in \mathcal{H}^t} h_k(t) \otimes \varepsilon_{\alpha_k}(t)\end{aligned} \tag{10}$$

where again $\varepsilon_{\alpha_k}(t) = \varepsilon(t)\exp\left(-j\alpha_k\theta(t)\right)$ and $h_k(t)\otimes\varepsilon_{\alpha_k}(t) = \int_{-\infty}^{+\infty} h(t-u)\varepsilon_{\alpha_k}(u)\mathrm{d}u$. Interestingly, the structure of a LPAV filter turns to be equivalent to the sum of LTI filtering operations on a transformed version of the signal. The architecture of a LPAV filter is illustrated in Fig. 1. The calculation of the filter coefficients, $h_k(t)$, is addressed in the next subsection.

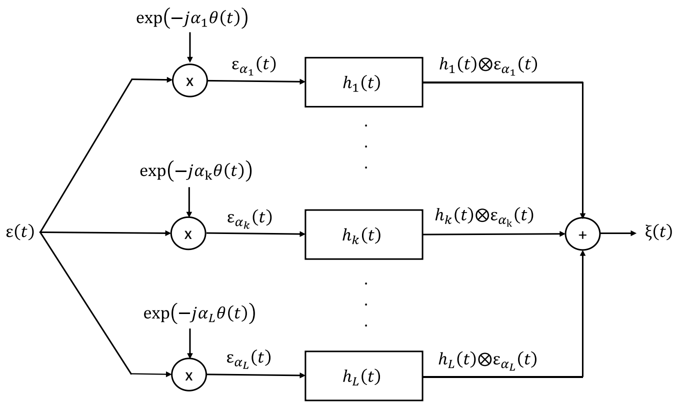

Fig. 1: The structure of a LPAV filter.

Calculation of the filter coefficients

Similarly to the CS case, if an angle-time cyclostationary (AT-CS) signal of interest $s(t)$ is to be extracted from its noisy measurement $x(t)$, the angle-time cyclic Wiener filter (AT-CWF) must then be designed as a LPAV filter whose own cyclic frequency set belong to that of the signal of interest. Under this condition, one can show that the following equation

$$S_{\tilde{x}_{\alpha_j}\tilde{x}_0}(f) = \sum_{k=1}^{K} H_k(f) S_{\tilde{x}_{\alpha_j}\tilde{x}_{\alpha_k}}(f) \tag{11}$$

holds for $1<j<K$, where K stands for the number of filter coefficients. These equations can be rewritten under matrix form:

$$\boldsymbol{H}(f) = \boldsymbol{\mathcal{L}}^{-1}(f)\boldsymbol{b}(f). \tag{12}$$

where $\boldsymbol{H}(f) = [H_1(f) \ldots H_K(f)]^T$ and $\boldsymbol{b}(f) = \left[S_{\tilde{x}_{\alpha_1}\tilde{x}_0}(f) \ldots S_{\tilde{x}_{\alpha_K}\tilde{x}_0}(f)\right]^T$ are is a $K \times 1$ vector (^{T} is the transpose operator), $\boldsymbol{\mathcal{L}}(f)$ is a $K \times K$ matrix such that $\boldsymbol{\mathcal{L}}_{m,n}(f) = S_{\tilde{x}_{\alpha_n}\tilde{x}_{\alpha_m}}(f)$. Note that $\boldsymbol{\mathcal{L}}(f)$ is a singular matrix so its pseudo-inverse returns the least mean square solution of this system. Interestingly, in the constant speed case CWF boils down to the classical CWF.

Application to bearing diagnostic

The application in this paper aims at extracting the vibratory component of a faulty rolling element bearing operating under variable speed conditions. The bearing has an inner race fault with characteristic fault frequency equal to 9.47 times the rotating frequency. The data are publicly available and more information on the system can be found in Ref. [5]. Since the inner race fault signature is expected to appear at the harmonics of the inner race fault and sidebands spaced by the shaft frequency, the cyclic frequencies (respectively orders) of the CWF (respectively AT-CWF) includes then (i) the fundamental shaft average frequency (order), (ii) the fundamental inner-race average frequency (respectively orders) values and (iii) its second harmonic. The negative frequencies (orders) of the latter are surely considered. The window length of the Welch estimator involved in both algorithms is set equal to 2^{10}. The speed profile of the signal, the temporal representation of the signal, the CWF-based and AT-CWF-based estimate are displayed in Fig. 2. As expected, the CWF-based estimate is way weaker than the AT-CWF-based estimate indicating a misestimation of the bearing fault component (see the close-ups in Fig. 3 for a more relevant interpretation). On the contrary, the vibratory fault component estimated by the AT-CWF is clearer revealing clear repetitive transients related to the fault. The application of the AT-CWF makes relevant the interpretation of the bearing component, enhancing its diagnosis and prognosis.

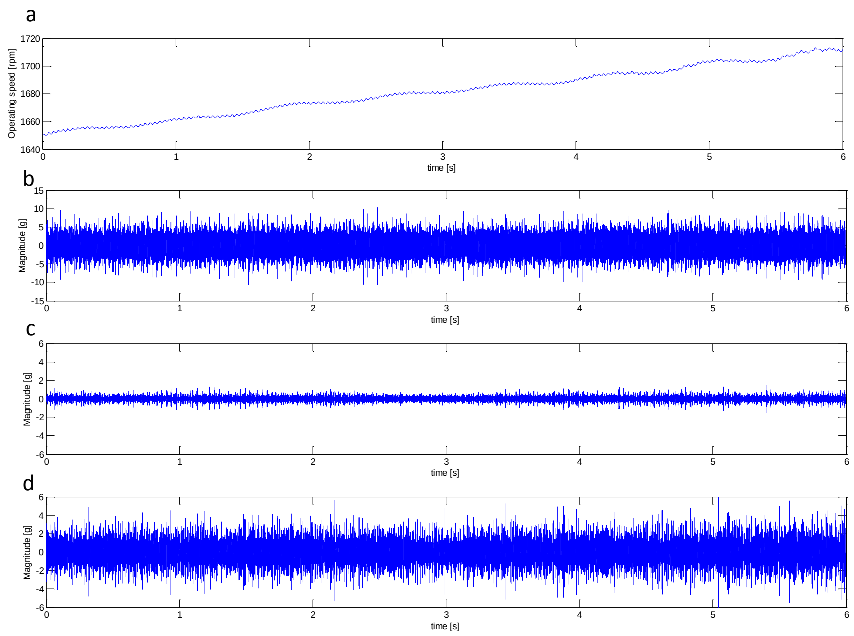

Fig. 2: (a) Speed profile in rpm, (b) the centred signal, (c) CWF-based and (c) AT-CWF-based estimate of the bearing fault vibratory component. Note that the scales in plots (c) and (d) are purposely kept the same for a proper comparison of the waveform energy.

Conclusion

This paper has addressed the problem of mechanical source separation when the system operates under fluctuating speed conditions. The problem has been theoretically formalized within the angle-time cyclostationary framework, offering a straightforward way to extend the cyclic Wiener filter to the variable speed conditions. The so-called angle-time cyclic Wiener filter turns to have a linear periodically angle-varying structure and its calculation is similar to that of the regular cyclic Wiener filter. The efficiency of the proposed approach was demonstrated on real vibration signals where the aim was to separate a fault vibratory component of rolling element bearing. The comparison between the classical and the proposed filter has asserted a clear superiority of the latter over the former. Such filtering is expected to enhance the fault detectability, diagnosis and prognosis of rolling element bearings.

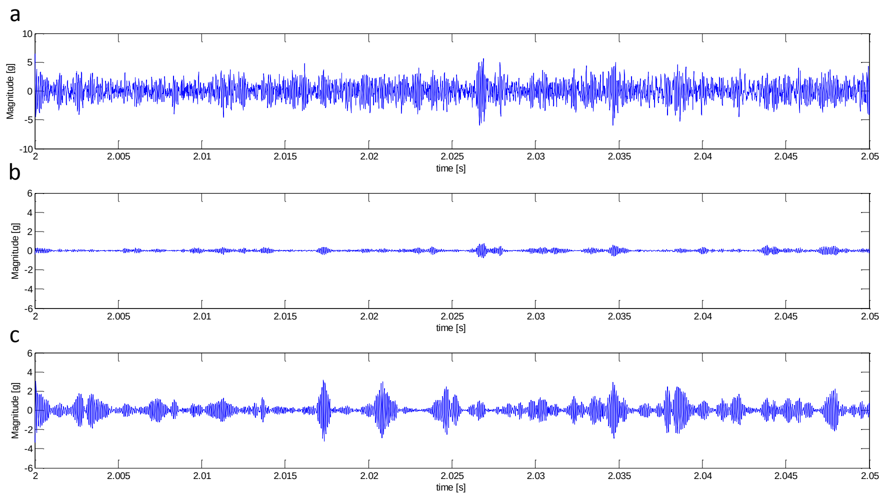

Fig. 3: (a), (b) and (c) are respectively the close-ups between 2 and 2.05s of plots (b), (c) and (d) in Fig. 2.

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