Novel approach for the estimation of transfer functions using a realistic dynamic model of gear and in-out zeros technique

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Abstract

The vibration signals of the rotating components in rotating machinery propagate through a transfer function on their way to the acceleration sensor. The transfer function distorts the shape of the vibration signal, damaging the ability to monitor the health status of the rotating components. There are limited techniques found in literature enabling to mitigate the transfer function effects. The current technique can estimate the transfer function magnitude without its original phase. This study presents a new concept enabling to estimate the transfer function with its phase for a gear's vibration signal. The new technique estimates the amplitude of the transfer function followed by the estimation of its poles and zeros under minimum-phase assumption, using autoregressive moving-average (ARMA) model and noise coloring. Then, the technique estimates the phase of the transfer function without minimum-phase assumption using a new approach, called in-out zeros, and a realistic dynamic model of a gear. The performance of the new concept is demonstrated using simulated transfer function and a simulated signals from dynamic model of a spur gear. A special experimental test rig was used in order to examine the new technique. The transfer function of the measured signals was estimated using the new technique, leading to the conclusion that the simulation or the technique for estimating the magnitude of the transfer function should be improved.

Keywords: transfer function estimation, minimum phase, ARMA model, poles and zeros.

Section 1: Introduction

Monitoring vibration signals is a widespread method for health monitoring ([1], [2]). The vibration signals are measured by accelerometers and are analysed by signal processing algorithms ([3-5]). A main challenge in the processing procedures is to mitigate the effects of the transfer function on the vibration signals ([6], [7]).

The spectrum signal of the vibration contains discrete frequencies, associated with the rotating components, and the background spectrum, associated with the magnitude of the transfer function, as depicted in Fig. 1 ([8], [9]). For gear vibration signals it is assumed that the magnitude of the transfer function can be approximated by the background, since wideband noise is generated by the teeth profile errors. The background can be estimated by several techniques, including Ceps-Lift ([10-13]), adaptive clutter separation (ACS, [8]) and autoregressive (AR) model ([14], [15]).

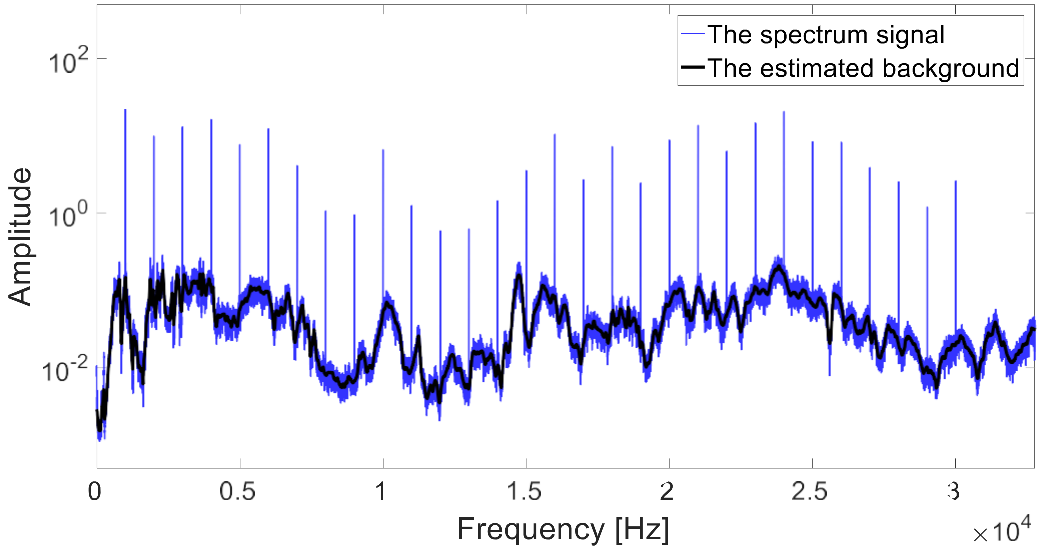

Fig. 1: An example of a spectrum signal and its estimated background

The phase of the transfer function cannot be approximated directly by the spectrum signal. The current approach for mitigating this challenge is to assume that the phase of the transfer function holds the minimum phase assumption, which states that the zeros and the poles of the transfer function are inside the unit circle, as explained in the theoretical background ([16]). However, this assumption does not hold for some transfer functions, as demonstrated in Section 2. In this study a novel technique is proposed for handling such cases.

The new technique described in Section 3 uses ACS for estimating the transfer function magnitude, and estimates the poles and zeros of the transfer function under minimum phase assumption by noise colouring and autoregressive moving-average (ARMA) model ([17], [18]). In the last step the phase is estimated by locating the zeros in several optional positions and finding the location minimizing the mean squared error (MSE, [19-21]) between the vibration signal and a reference gear simulation.

The technique is demonstrated in Section 4 on a simulated gear signal and a simulated transfer function. In Section 5 the technique is applied on measured data. The estimated transfer function does not explain the differences between the measured and the simulated signals probably because of the errors in the estimation of the magnitude of the transfer function and differences between the simulated and the measured signals.

Section 2: Theoretical background

ACS: ACS estimates the spectrum background by filtering extreme deviations. The algorithm separates the spectrum to consecutive segments and selects the median in each of them. In the last step the algorithm interpolates the selected values, resulting in an estimation of the background. An example of the estimated background by ACS is depicted in Fig. 1. The technique is described in detail in Ref. [8].

Minimum phase assumption: The poles of stable transfer functions are inside the unit circle in complex plane. Furthermore, if the inverse transfer function is also stable the zeros of the transfer function are also inside the unit circle, as depicted in Fig. 2. The minimum phase of the magnitude of a transfer function can be estimated by setting to zero the negative quefrencies and doubling the positive quefrencies, as described and explained in Ref. [16].

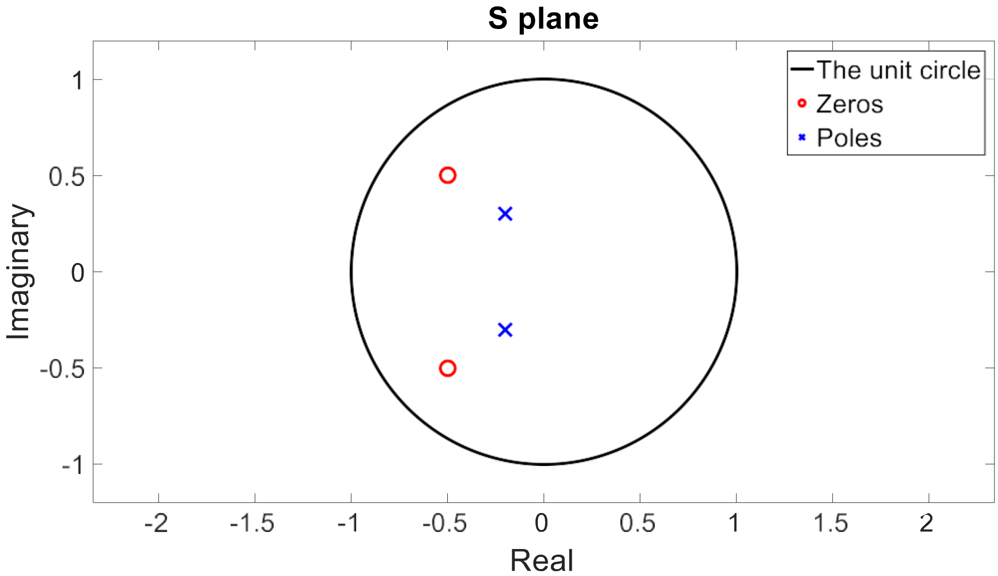

Fig. 2: An example of the zeros and pols of a minimum phase transfer function in S plane

Not all the transfer functions hold the minimum phase assumption. For example, we measured 24 transfer functions on several test rigs by hammer-taps measures ([22], [23]). Those transfer functions do not hold the minimum phase assumption - for example, a measured transfer function is depicted in Fig. 3. The negative quefrencies do not equal to zero, as presented in Fig. 3 (b). Thus, it can be concluded that the transfer function does not hold the minimum phase assumption ([16]).

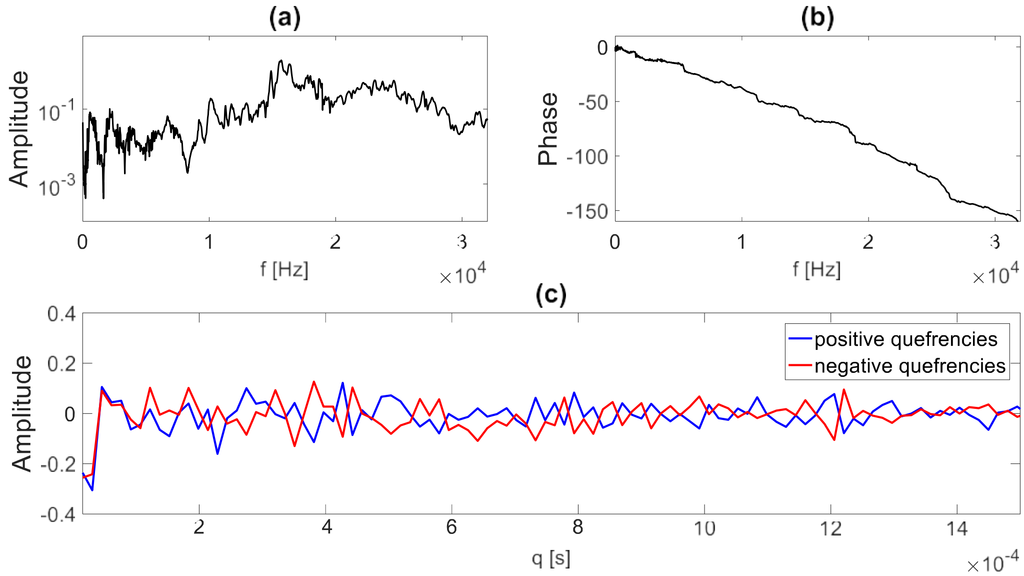

Fig. 3: An example of a measured transfer function. (a) The magnitude of the transfer function, (b) the phase of the transfer function and (c) the positive and negative quefrencies of the transfer function

<u>ARMA model:</u> ARMA model fits the noise behavior to Eqn 1 ([17], [18]). It estimates the coefficients that explain the statistical distribution of the noise. These coefficients correspond to the zeros and poles of the transfer function.

$$a_n y[n]+\ldots\ a_{n-k} y[n-k] = b_n x[n]+\ldots\ b_{n-k} x[n-k] \qquad (1)$$

<u>Possible positions of the zeros:</u> When the magnitude of the transfer function is known, the possible locations of the zeros are limited. For a zero without an imaginary component, its possible location is inverse proportionally to the unit circle. For a zero with an imaginary

component, its possible location is inverse proportional to the unit circle together with its conjugate zero ([16]). An example is depicted in Fig. 4.

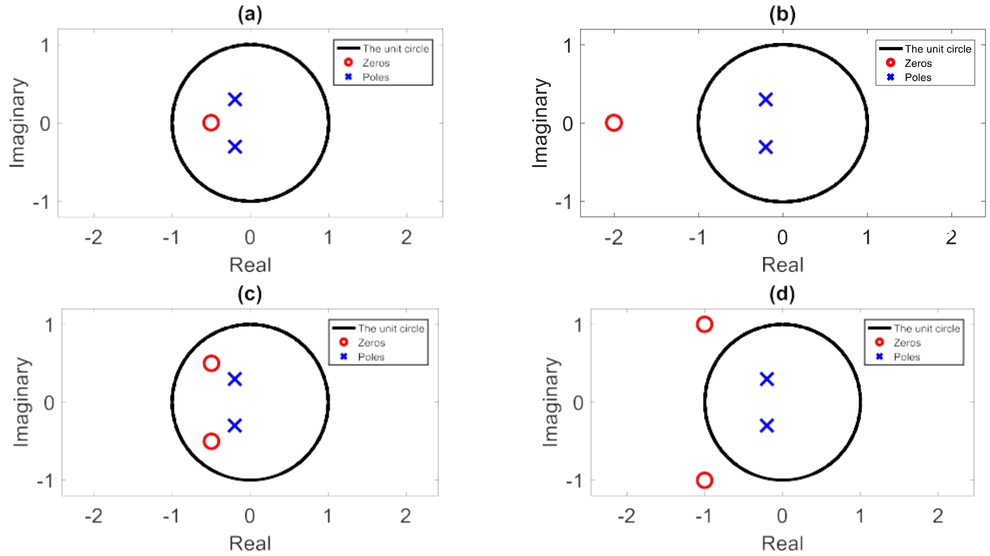

Fig. 4: An example of inverse zeros. (a) A zero inside the unit circle, (b) the inverse proportion of the zero from (a), (c) a pair of zeros inside the unit circle and (d) the inverse proportion of the pair of zeros from (c)

Section 3: Transfer function estimation

The new technique estimates the transfer function by 4 steps as described in Diagram 1. In the first step the transfer function's magnitude is approximated by the spectrum background, estimated by ACS. In the second step the approximated magnitude is multiplexed with white noise, and the signal is converted to the time domain. In the third step the zeros and poles of the transfer function are estimated by ARMA model. In the last step, the phase is estimated by locating the zeros in all of their possible positions, and the phase is selected to minimize the MSE between the model from Ref. [24] and the measured signal.

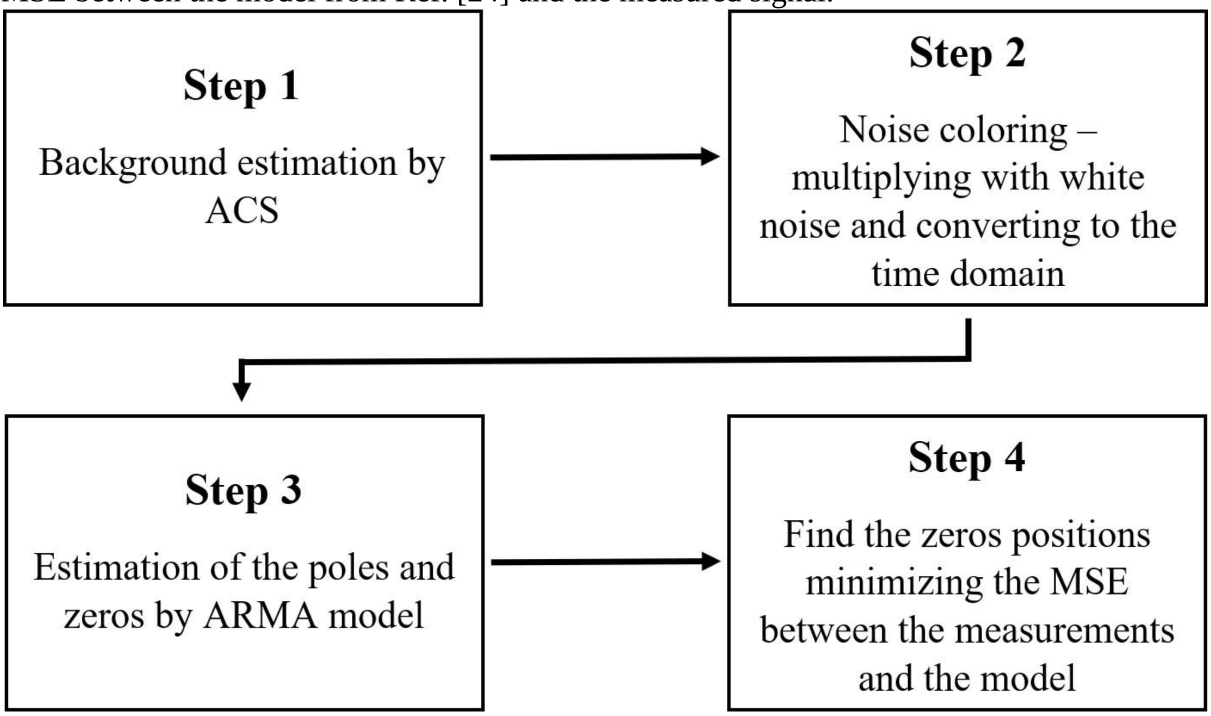

Diagram 1: Block diagram of the new algorithm for estimating the transfer function phase

The degree of the ARMA model can be selected by searching the minimal degrees of the poles and the zeros such that the estimated magnitude of the transfer function fits the estimated background. The ACS parameters can be selected as explained in Ref. [8].

Section 4: Demonstration on simulated data

The simulated signal is composed of the simulated transfer function depicted in Fig. 5, and a gear simulation from Ref. [24] with white noise. The synchronous average signal ([24]) of the latter is depicted in Fig. 6 (a). The estimated transfer function is presented in Fig. 5, together with the original transfer function. The estimated phase fits the original phase, as opposed to the estimated phase using the minimum phase assumption.

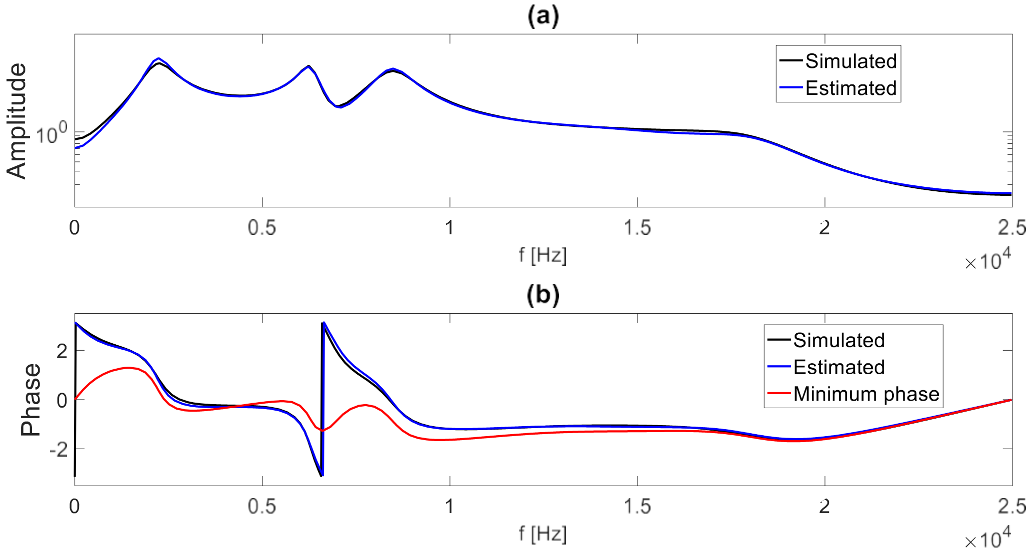

Fig. 5: The simulated transfer function and its estimations by the new technique (blue) and using minimum phase assumption (red). The graph of the estimated transfer function by the new technique conceals some of the segments of the simulated transfer function. (a) – magnitude of the transfer functions, (b) – phase of the transfer functions.

The ACS segment size was 50 bins. The degrees of the coefficients of a and b in the ARMA model were 10 and 5, respectively.

Section 5: Demonstration on measured data

A vibration signal of a gear was measured in the study, its synchronous average is depicted in Fig. 6 (a) bedsides the simulated signal. The defected tooth is depicted in Fig. 6 (b). The new technique was applied on the measured signal of the gear.

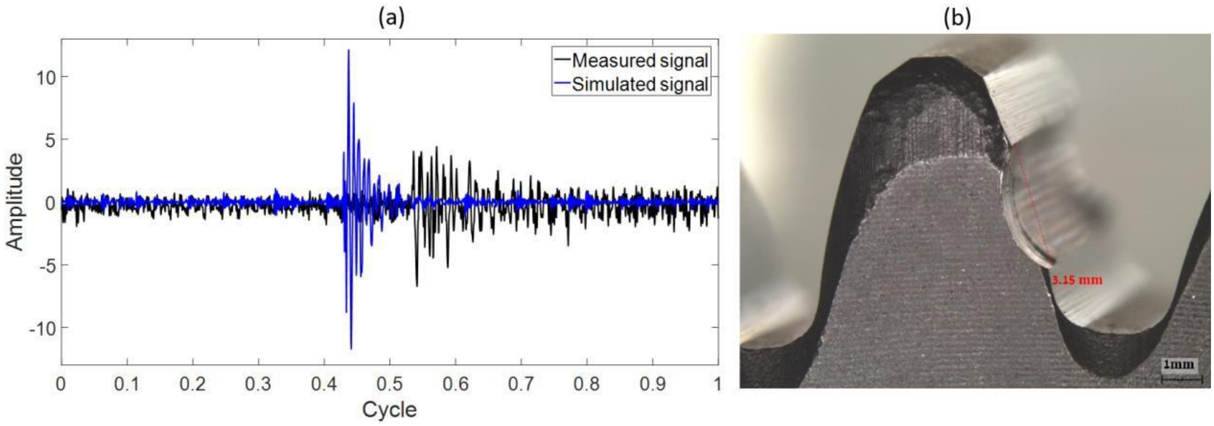

Fig. 6: (a) an example of a simulated signal from Ref. [24] and a measured signal with a full tooth face fault as depicted in (b).

The estimated transfer function is depicted in Fig. 7. The estimated signal after the transfer function is presented in Fig 7 (c).

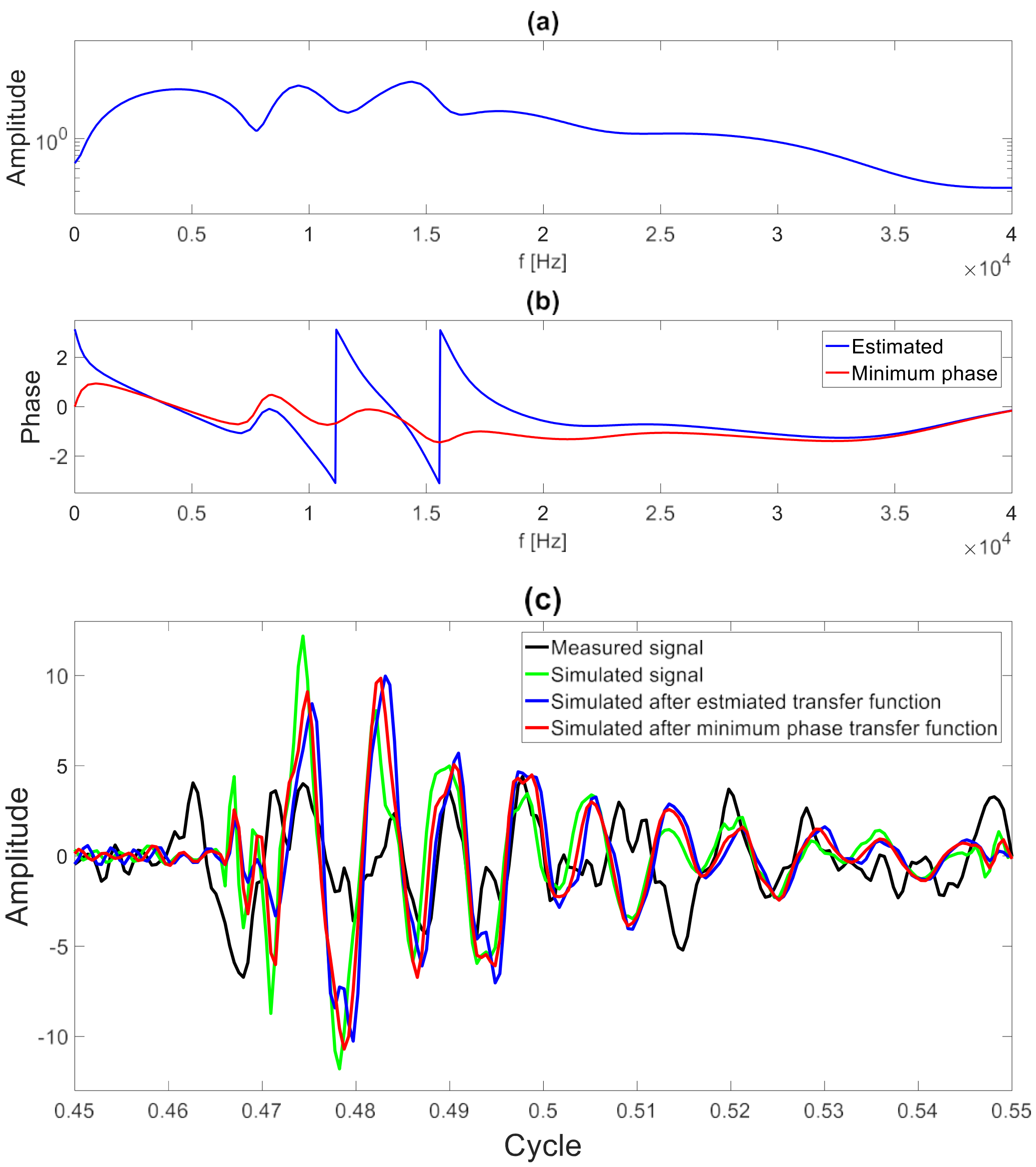

Fig. 7: (a) estimated magnitude of the transfer function, (b) estimated phase of the transfer function by the new technique (blue) and using minimum phase assumption (red), (c) the measured signal (black) and the simulated signal (green) together with the simulated signal

after the estimated transfer function by the new technique (blue) and using minimum phase assumption (red).

The MSE between the original simulation and the measured signals is 1.27. However, the MSE between the simulated signal after the estimated transfer function by the new technique and using minimum phase assumption are 1.24 and 1.26, respectively. The estimated transfer function decreases the MSE, but just by a small amount. The estimation of the phase by the new technique decreases the MSE in comparison to the estimated phase using minimum phase assumption, again by a small amount.

We believe that the estimated magnitude of the transfer function is far from the original one and that the simulated signal has significant difference in comparison to the measured signal. These factors prevent a more accurate demonstration of the simulated signal after the transfer function. However, even in this case the estimation of the phase by the new technique improves the decrease in the MSE from 0.01 to 0.03 in comparison to the estimated phase using minimum phase assumption. To conclude, the gear simulation or the estimation of the magnitude of the transfer function or both should be improved.

Section 6: Summary and conclusions

In this study a new technique for estimating the transfer function phase is presented. The technique uses ACS, ARMA model and in-out zeros procedure.

The technique was demonstrated on simulated vibration signal of a gear and a simulated transfer function. The estimated transfer function resembled the original one. The new technique was applied on a measured signal of a gear, leading to the conclusion that the simulation or the estimation of the magnitude of the transfer function should be improved.

The new technique enables the estimation of the phase of transfer function without holding the minimum phase assumption. The concept of the new technique may help to further develop of the abilities to estimate transfer functions.

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