Student Paper ☒

Multi-Impact Force Identification on Aircraft Composite Structures using Operational Modal Analysis

Morteza Payab [1], Adrian C. Orifici [1], Akbar A. Khatibi [1,*]
[1] *School of Engineering, RMIT University, GPO Box 2476, Melbourne, Victoria 3001, Australia*

Abstract

This research focuses on reconstructing the impact forces applied to a composite panel using vibrational responses in the presence of environmental noise from wind loads. Sparse regularisation is introduced for impact-force reconstruction, considering the sparse nature of impact-force in the time domain. A monotonic two-step iterative shrinkage/thresholding (MTWIST) algorithm based on minimising l_1-norm is developed to solve a class of convex optimisation problems in a highly underdetermined "impact-force reconstruction" model with an ill-conditioned measurement matrix. In the sparsity frame, the proposed sparse regularisation method determines the actual simultaneous impacts' locations from many candidate sources and reconstructs the time and magnitude of the impact forces simultaneously. Simulations are conducted on a simply supported rectangular plate in the presence of a high noise level to illustrate the effectiveness and applicability of the MTWIST algorithm. Results demonstrate the efficacy of the proposed method in solving the inverse problem of multi-impact force identification.

Keywords: Impact-force reconstruction, Sparse regularisation, Experimental modal analysis, Iterative shrinkage/thresholding algorithm

Introduction

Impact-force identification is a vital engineering issue in aircraft structural health monitoring (SHM), leading to modified performance evaluation, design optimisation, noise suppression and vibration control. Since direct measurement of impact forces on aircraft structures is complex, inverse methods, using outputs induced by impact **[1],** can be used to evaluate those forces. Force identification by inverse methods remains a challenging problem since fewer measured responses than the number of impact sources make these methods under-determined. Additionally, a small error in measured responses can lead to a significant fluctuation in the desired solution. Regularisation methods must be utilised to transform such an ill-posed problem into a well-posed problem **[2-3]**. The classical l_2-norm-based regularisation methods such as Tikhonov **[4-6]** and truncated singular value decomposition (TSVD) methods **[7- 9]** cannot determine the impact location and simultaneously reconstruct its time history. Impact localisation is a much more complex issue **[8]** and requires more sensors **[1]**, while in many classical l_2-norm-based regularisation methods, the number of measurements must not be less than the number of unknown sources. In contrast, the sparse regularisation methods using l_1 penalty requires the solution to have minimum nonzero values and yield a very sparse solution **[10]**.

Despite many studies being carried out in the force identification field, a comprehensive review shows that simultaneous impacts have not received much attention. In this paper, a general sparse regularisation model based on minimising l_1-norm is proposed to reconstruct the impact force, which simultaneously identifies the location, magnitude and time history of multiple simultaneous impacts. A monotonic two-step iterative shrinkage/ thresholding

(MTWIST) algorithm is proposed to find the sparse solution to the underdetermined model from incomplete and inaccurate measurements.

Impact-force sparse reconstruction formulation

Force reconstruction for the single-source case can be modelled as a convolution integral of the excitation force $f(t)$ with the impulse response function (IRF), defined as:

$$y(t) = h(t) \otimes f(t) = \int_0^t h(t-\tau) f(\tau) \mathrm{d}\tau \tag{1}$$

where $\otimes$ denotes the convolution operation and τ is the time-delayed operation satisfying $t \geq \tau$. The continuous convolution model should be discretised numerically for computing as follows:

$$y(j\Delta t) = \Delta t \sum_{i=1}^{n} h\big((j-i+1)\Delta t\big) f(i\Delta t) \tag{2}$$

where Δt is the time interval, i, j are counters, and n is the data length of the discrete IRF. The compact matrix-vector form of Eq. (2) after discretisation can be rewritten as y = Hf where the transfer matrix H∈R$^{n\times n}$ contains the system dynamic characteristics and depends on excitation and measurement locations.

Force reconstruction for a multiple-input multiple-output (MIMO) system can then be written as:

$$\begin{bmatrix} \mathrm{y}_1 \\ \mathrm{y}_2 \\ \vdots \\ \mathrm{y}_M \end{bmatrix} = \begin{bmatrix} \mathrm{H}_{11} & \mathrm{H}_{12} & \cdots & \mathrm{H}_{1N} \\ \mathrm{H}_{21} & \mathrm{H}_{22} & \cdots & \mathrm{H}_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{H}_{M1} & \mathrm{H}_{M2} & \cdots & \mathrm{H}_{MN} \end{bmatrix} \begin{bmatrix} \mathrm{f}_1 \\ \mathrm{f}_2 \\ \vdots \\ \mathrm{f}_M \end{bmatrix} \tag{3}$$

where the integer numbers M and N denote the number of the measurement sensors and the candidate sources, respectively. Submatrix H_{ij} is the transfer matrix between the response point i and the excitation point j. Note that H_{ij} have the condition number in order of 1×10^{17} to 1×10^{19} and finding a stable solution is not guaranteed. The element expansion of the governing equation Eq. (3) matrix can be determined as follows:

$$\begin{bmatrix} y(\Delta t) \\ y(2\Delta t) \\ y(3\Delta t) \\ \vdots \\ y(n\Delta t) \\ y((n+1)\Delta t) \\ y((n+2)\Delta t) \\ \vdots \\ y((m-2)\Delta t) \\ y((m-1)\Delta t) \\ y(m\Delta t) \end{bmatrix}_{m\times1} = \Delta t \begin{bmatrix} h_e(\Delta t) & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ h_e(2\Delta t) & h_e(\Delta t) & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ h_e(3\Delta t) & h_e(2\Delta t) & h_e(\Delta t) & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ h_e(n\Delta t) & h_e((n-1)\Delta t) & h_e((n-2)\Delta t) & \cdots & h_e(\Delta t) & 0 & 0 & \cdots & 0 \\ 0 & h_e(n\Delta t) & h_e((n-1)\Delta t) & \cdots & h_e(2\Delta t) & h_e(\Delta t) & 0 & \cdots & 0 \\ 0 & 0 & h_e(n\Delta t) & \cdots & h_e(3\Delta t) & h_e(2\Delta t) & h_e(\Delta t) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & h_e(n\Delta t) & h_e((n-1)\Delta t) & h_e((n-2)\Delta t) & \cdots & h_e(\Delta t) \end{bmatrix}_{m\times m} \times \begin{bmatrix} f(\Delta t) \\ f(2\Delta t) \\ f(3\Delta t) \\ \vdots \\ f(n\Delta t) \\ f((n+1)\Delta t) \\ f((n+2)\Delta t) \\ \vdots \\ f((m-2)\Delta t) \\ f((m-1)\Delta t) \\ f(m\Delta t) \end{bmatrix}_{m\times1} \tag{4}$$

This work focuses on a problem where only a single accelerometer is employed for measurement, and nine candidate sources are selected. This case presents a severely underdetermined ill-posed problem. The minimum l_1-norm solution of Eq. (4) in a highly underdetermined system can be imposed by the following optimisation

$$\underset{\mathrm{f}}{\text{minimise}} \quad \|\mathrm{f}\|_1 \; ; \qquad \text{subject to } \; \mathrm{y}=\mathrm{Hf} \tag{5}$$

where $\|\mathrm{f}\|_1$ denotes the l_1-norm, i.e., the sum of the absolute values of all the components of f. In practice, the measured response vector, y, is commonly inaccurate or contaminated by noise. The equality condition in Eq. (5) is transformed to the inequality form as

$$\underset{\mathrm{f}}{\text{minimise}} \quad \|\mathrm{f}\|_1 \; ; \qquad \text{subject to } \; \|\mathrm{Hf}-\mathrm{y}\|_2^2 \leq \delta \tag{6}$$

where δ denotes the noise bound. Eq. (6) can be written as a minimiser of a convex unconstrained objective function

$$\underset{\mathrm{f}}{\text{minimise}} \quad G(\mathrm{f})=\frac{1}{2}\|\mathrm{Hf}-\mathrm{y}\|_2^2 + \lambda\|\mathrm{f}\|_1 \tag{7}$$

where $G(\mathrm{f})$ is a convex and non-differentiable function, and $\lambda \in [0, +\infty)$, the regularisation parameter, can be tuned. For Tikhonov regularisation, the l_2-norm solution tends to zero, as $\lambda \to +\infty$ while for the l_1-norm regularisation, the solution f_{l1} is always zero, when **[14]**

$$\lambda \geq \lambda_{\max} = \|2\mathrm{H}^{\mathrm{T}}\mathrm{y}\|_{\infty} \quad (8)$$

where $\|\cdot\|_{\infty}$ denotes the l_{∞}norm. Therefore the l_1-norm regularisation typically yields a sparse solution. An MTWIST algorithm is proposed for solving the l_1-norm regularisation of impact-force sparse reconstruction in a noninvertible case **[12]**. An overview of the proposed method is shown in Fig. 1.

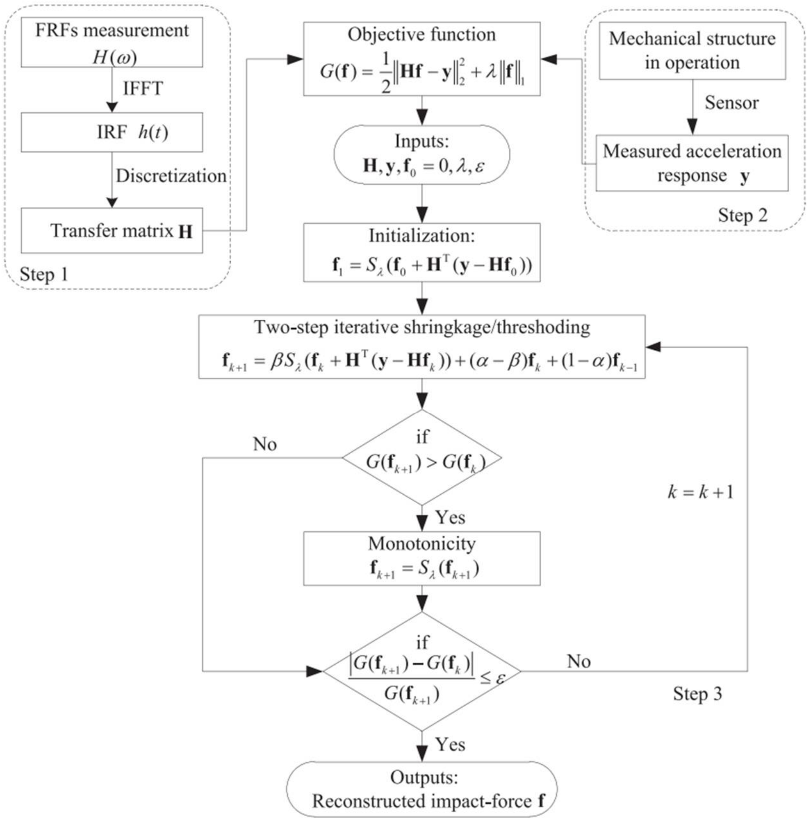

Fig. 1. The MTWIST algorithm for impact-force sparse reconstruction.

Simulation Study

The impact force acting on a thin, simply-supported composite plate is reconstructed by finding the impact location, magnitude and time history. The spatial distribution over the plate structure is multiple-input single-output (MISO) as there is only a single accelerometer measurement. Three different MISO cases are considered under different noise levels, and in all these cases, at least two simultaneous impacts are applied to the structure. The magnitude of impact force is assessed by the peak relative error between the measured force, $\mathrm{f}_{\mathrm{measured}}$, and the reconstructed force, $\mathrm{f}_{\mathrm{reconstructed}}$, defined as:

$$\text{Peak relative error} = \frac{|\max(\mathbf{f}_{\mathrm{measured}}) - \max(\mathbf{f}_{\mathrm{reconstructed}})|}{|\max(\mathbf{f}_{\mathrm{measured}})|} \times 100\% \quad (9)$$

Table 1 gives the simulation parameters. Note that different from previous studies **[7,11]** where structural damping was set as $\eta = 0.03$, here $\eta = 0.003$. In general, a lightly damped structure is far more difficult for force identification. The simulation reveals that the smaller the structural damping, the worse the ill-posedness of the inverse problem. Impact force distribution and accelerometer location is produced by discretising the space with six gridlines over the structure, as shown in Fig. 2. Three different impact scenarios were examined. The first scenario has two impact locations (simultaneous impact). In comparison, the second scenario has three impact locations (two of which are concurrent), and the third scenario has four impact locations (grouped into two sets of two simultaneous impacts). Four noise levels of 10%, 20%, 30% and 40% are considered, which respectively correspond to noise-to-signal ratios of the measured response $\|\mathbf{e}\|_2/\|\mathbf{y}\|_2$ of 5.82%, 11.54%, 17.47% and 23.29%.

Table 1: Simulation parameters of the simply supported rectangular plate

Parameters	**Values**
Plate Length a	0.6 m
Plate Width b	0.5 m
Plate Thickness h	0.0015 m
Structural Damping η	0.003
Poisson Ratio ν	0.3

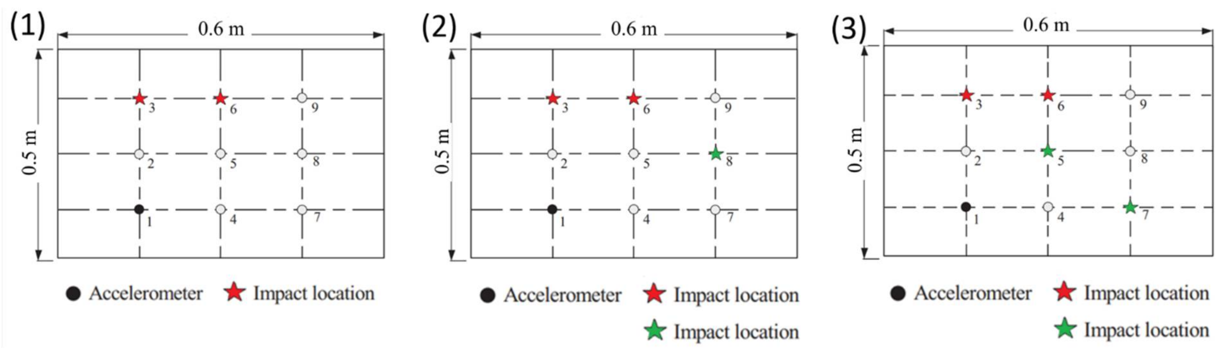

Fig. 2. (1) First scenario; (2) Second scenario; (3) Third scenario

The impact-force identification results using the proposed MTWIST algorithm for the 3rd scenario is illustrated in Fig. 3, and the outcome for the other two cases are similar. As shown in Fig. 3, all impact sources at different locations are clearly distinguished for the four noise levels. The reconstructed force values in other assumed locations are small enough to be ignored.

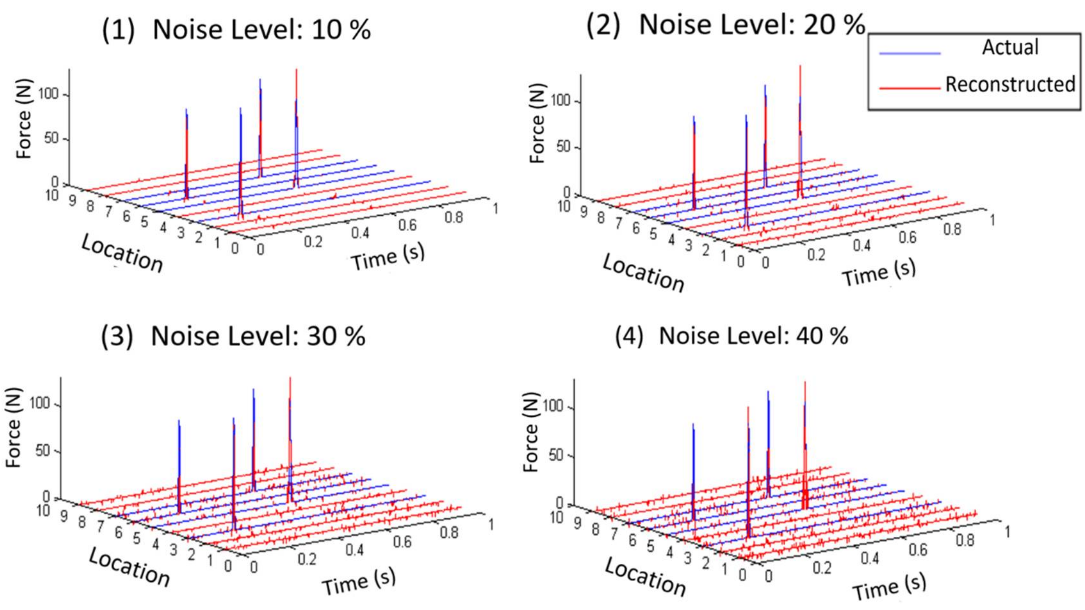

Fig. 3. Results for the 3rd scenario with noise levels: (1) 10%; (2) 20%; (3) 30%; (4) 40%

For the 3rd scenario, the force-time histories around the peak load for locations 3, 6, 5 and 7 with four noise levels are depicted in Fig. 4. The force-time curves from the numerical solutions are consistent with the measured force, and the solution accuracy decreases with the increasing noise level. Similar outcomes were seen for the other two scenarios. Results across the three scenarios show that the largest time history relative error among the four noise levels in any scenario was 0.3%.

The relative error in evaluating the peak forces for the first scenario with two simultaneous impacts was 1.2% and 21.3% for noise levels of 10% and 40%, respectively. The same trend was observed with the other two scenarios in which increasing the noise level resulted in higher relative error values. It is noteworthy that the relative error in the second scenario at 40% noise was more significant than that in the other two cases studied in this paper. It is postulated that the location of impact points in respect to plate mode shapes may contribute to such high disagreement between evaluated and actual peak force. Further studies are undertaken to characterise the possible interaction of mode shapes and nominated impact points.

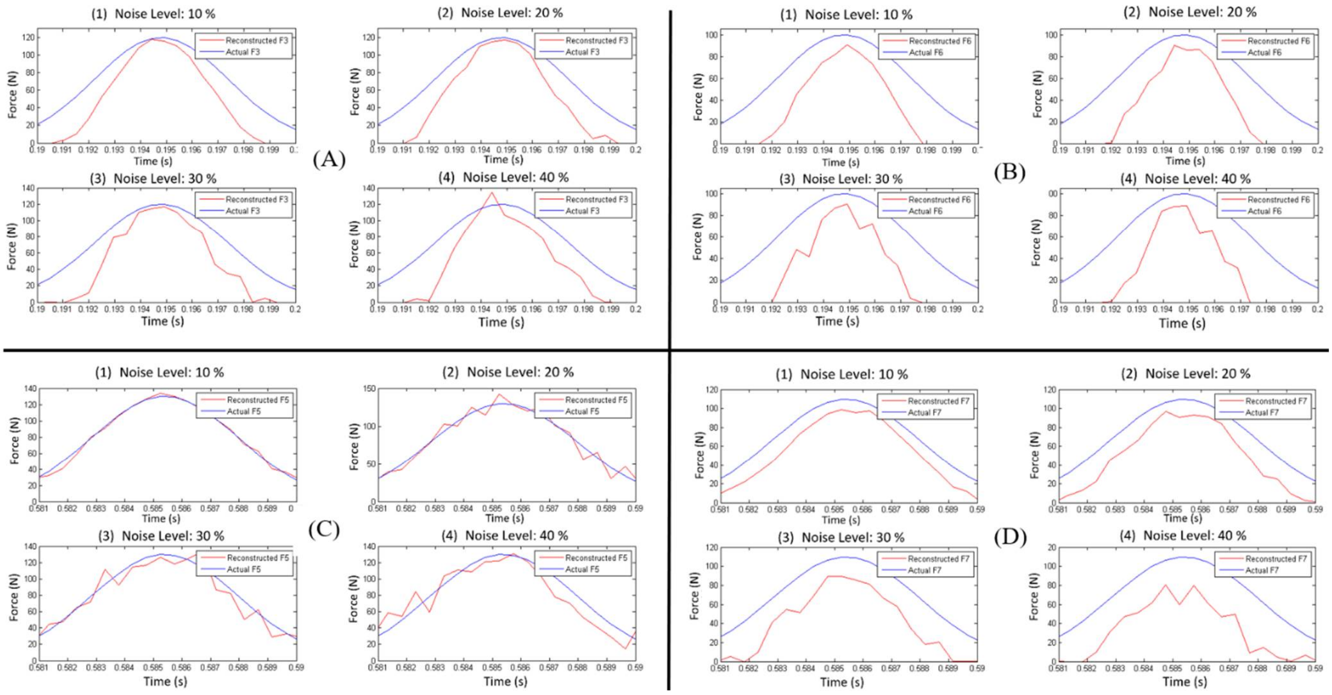

Fig. 4. The zoomed-in results of reconstruction the force acting on 3rd point (A), 6th point (B), 5th point (C) and 7th point (D) with different noise levels: (1) 10%; (2) 20%; (3) 30%; (4) 40%

Conclusions

Here, we introduced and demonstrated the application of sparse regularisation to impact-force identification, a particular category of force reconstruction. First, taking into account the sparse characteristic of impact-force in the time domain, l_1-norm sparse regularisation for solving an underdetermined system of impact-force reconstruction is introduced. Second, MTWIST is successfully developed for impact-force sparse reconstruction from highly incomplete and inaccurate measurements where the impact location, magnitude and time history of simultaneous impact-forces are determined concurrently. Finally, the proposed sparse regularisation method was examined under different scenarios by which the following results are concluded:

- The location and time histories of impact forces in all scenarios were determined among nine candidate sources with high accuracy, using one accelerometer in the presence of environmental noise.
- The magnitude peak relative errors show that increasing the number of impacts does not necessarily reduce the accuracy of the reconstruction results.
- Simulation proved that the MTWIST algorithm has good reconstruction performance and is not very sensitive to noise. Therefore, sparse regularisation as an alternative technique has advantages in identifying the impact force from accuracy, robustness and computation efficiency points of view.

References

1. B. Qiao, X. Zhang, J. Gao, X. Chen, *Impact-force sparse reconstruction from highly incomplete and inaccurate measurements*, J. Sound Vib. 376 (2016) 72-94.
2. Khoo, S., et al., *Impact force identification with pseudo-inverse method on a lightweight structure for under-determined, even-determined and over-determined cases.* International Journal of Impact Engineering, 2014. **63**: p. 52-62.
3. Qiao, B., et al., *Sparse regularisation for force identification using dictionaries.* Journal of Sound and Vibration, 2016. **368**: p. 71-86.
4. Thite, A. and D. Thompson, *The quantification of structure-borne transmission paths by inverse methods. Part 2: Use of regularisation techniques.* Journal of Sound and Vibration, 2003. **264**(2): p. 433-451.
5. Renzi, C., C. Pezerat, and J.-L. Guyader, *Vibratory source identification by using the finite element model of a subdomain of a flexural beam.* Journal of Sound and Vibration, 2013. **332**(3): p. 545-562.
6. Liu, Y. and W.S. Shepard, *Reducing the impact of measurement errors when reconstructing spatial dynamic forces.* Journal of vibration and acoustics, 2006. **128**(5): p. 586-593.
7. Thite, A. and D. Thompson, The quantification of structure-borne transmission paths by *inverse methods. Part 1: Improved singular value rejection methods.* Journal of Sound and Vibration, 2003. **264**(2): p. 411-431.
8. Inoue, H., J.J. Harrigan, and S.R. Reid, *Review of inverse analysis for indirect measurement of impact force.* Applied Mechanics Reviews, 2001. **54**(6): p. 503-524.
9. Qiao, B., et al., *A novel method for force identification based on the discrete cosine transform.* Journal of Vibration and Acoustics, 2015. **137**(5): p. 051012.
10. Tropp, J.A. and S.J. Wright, *Computational methods for sparse solution of linear inverse problems.* Proceedings of the IEEE, 2010. **98**(6): p. 948-958.
11. Figueiredo, M.A., R.D. Nowak, and S.J. Wright, *Gradient projection for sparse reconstruction: Application to compressed sensing and other inverse problems.* IEEE Journal of selected topics in signal processing, 2007. **1**(4): p. 586-597.
12. Bioucas-Dias, J.M. and M.A. Figueiredo, *A new TwIST: two-step iterative shrinkage/thresholding algorithms for image restoration.* IEEE Transactions on Image processing, 2007. **16**(12): p. 2992-3004.
13. D.L. Donoho, *For most large underdetermine systems of linear equations the minimal l_1-norm solution is also the sparsest solution*, Communications on Pure and Applied Mathematics 59 (7) (2006) 797–829.
14. S.J. Kim, K. Koh, M. Lustig, S. Boyd, D. Gorinevsky, *An interior-point method for large-scale l_1-regularised least squares*, IEEE Journal of Selected Topics in Signal Processing 1 (4) (2007) 606–617.