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New applications of cepstrum analysis in machine diagnostics

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Abstract

A recent research project has confirmed much of the promise of cepstrum analysis in its application to machine diagnostics, involving analysis of response signals. The diagnostic information is usually clearest in the forcing functions, but measurements are always modified by transfer functions (TFs), different for each path. The cepstrum is useful as a precursor to operational modal analysis (OMA), where it can be used to remove or enhance modal properties or shaft speed related components, even for varying speed. The cepstrum also provides an alternative OMA method, because for single input/multiple output (SIMO) systems, the modal information can be fitted in a pole/zero model. The paper gives examples of a number of areas in which cepstrum analysis has given rise to unique processing techniques, so far not achievable by other means. One is by application of comb lifters in the angle cepstrum domain to remove everything except the signals from a particular gear pair, with all transfer functions removed, giving a signal closely related to the static transmission error (STE) exciting the responses. In the second case, the gear signals masking the bearing signals are removed by a comb notch lifter, leaving a broadband signal whose resonances carry the bearing fault information. The other application uses the cepstral OMA technique to determine the TF from the gearmesh to the measurement point, based on the broadband signal at the mesh (from tooth sliding etc). An inverse filter, generated from the TF, is then applied to the discrete frequency harmonics of each gear, to generate waveforms similar to the forcing functions, and thus independent of speed. This has now been demonstrated for TE signals (converting them from dynamic to static) and for acceleration signals from a single stage gearbox.

Keywords: Cepstrum analysis, machine diagnostics, operational modal analysis, transfer functions, inverse filtration, forcing functions

Introduction

The cepstrum is the inverse Fourier (or z-) transform of the log spectrum, and has two very important properties allowing two types of separation in vibration response signals: 1) It can separate signals from different sources, in particular those related to shaft speeds in the angle/order domains, and those carried by broadband resonances in the time/frequency domains, and 2) It can be used to separate forcing and transfer functions to different measurement points from the same source.

The complex cepstrum [1] is defined in Eq. (1):

$$C_c(\tau)=\Im^{-1}\left\{\log\left(F(f)\right)\right\}=\Im^{-1}\left\{\ln\left(A(f)\right)+j\phi(f)\right\} \tag{1}$$

The first proposers of the cepstrum (in a different form) coined a number of terms by reversal of the first syllable (including “cepstrum” from “spectrum”), such as “quefrency” from “frequency” as the x-axis of the cepstrum (actually time, as for the autocorrelation function), “rahmonics” from “harmonics”, as equally spaced discrete delta functions in the cepstrum, and “lifter” from “filter”.

For SIMO (single input, multiple output) systems, the cepstrum of a response is the sum of the cepstra of the forcing and transfer functions, as the Fourier transform converts the time domain convolution to a product in the spectrum, and the logarithm to a sum in both the log spectrum and cepstrum. The complex cepstrum is thus reversible to the time functions, but only for transients, whose phase can be unwrapped to a continuous function of frequency. It cannot be applied to multiple input, multiple output (MIMO) responses which are a sum of convolutions. Oppenheim and Schafer, [2], generated the analytical form of the complex cepstrum in the z-plane, showing that for minimum phase functions it is causal, so that the log amplitude and phase of transfer functions are related by a Hilbert transform. In that case, the complex cepstrum can be generated from the log amplitude spectrum only, without the need to measure and unwrap the phase. The so-called real cepstrum is obtained by setting the phase in Eq. (1) to zero. Many mechanical systems have minimum phase properties, so the real cepstrum can then be used for OMA where only the spectrum amplitude can be measured. Whenever the forcing function is broadband, either impulsive or broadband random, its cepstrum is at very low quefrency, and the transfer function can be fitted (for its poles and zeros) to the high quefrency part where it dominates.

When a pole-zero model is truncated above a certain frequency, poles and zeros are correct, but the residues are wrong. They can be corrected by an equalisation function that is primarily dependent on the ratio of the numbers of poles and zeros outside the measurement band, but not very sensitive to their actual placement. It is independent of the colour of the forcing function. A detailed discussion on the generation of the equalisation function is given in [3].

Applications

Figure 1 compares updated (OMA) and measured (EMA) FRFs from a simulated gearbox signal [4] with speed varying by ±5%. The former was for free-free conditions, neglecting the rigid body modes of the soft suspension in the measurement required to produce the free-free response [4]. The figure on the left shows (in red) the FRF obtained directly from the fitted

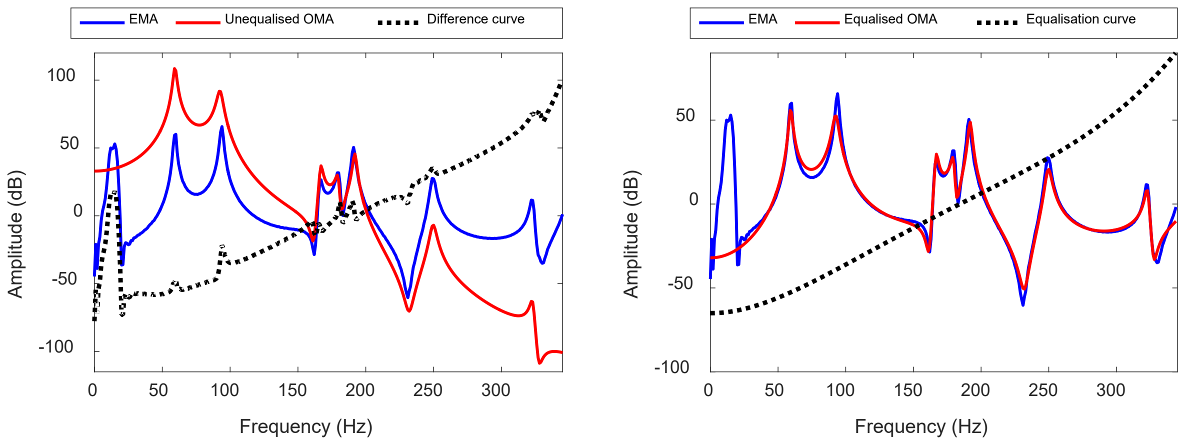

Figure 1: (a) Measured and unequalised regenerated FRFs with raw dB difference curve; (b) Measured and equalised regenerated FRFs with smoothed equalisation curve

poles and zeros, without equalisation, showing the effects of out-of-band terms; the plot on the right shows the vast improvement obtained after equalisation.

Two different types of liftering have been found useful in machine diagnostics, viz. comb (and comb-notch) liftering to enhance (or remove) the additive (harmonic) or modulation (sideband) components in response signals, and exponential liftering to separate the low quefrency part of the response cepstrum, usually dominated by the modal properties of the structure, from the high quefrency part, including discrete harmonics and sidebands, often indicative of the intrinsic forcing functions, without the influence of the modal properties, and thus more independent of speed variations.

Figure 2 shows examples from the same case as Fig. 1 for the use of an exponential lifter to remove the constant and varying speed excitations from the simulated gear forcing signal, adding only a small amount of extra damping, which could be subtracted after curve fitting. The broadband signals exciting the modes over the full frequency band come from the sliding action of the gear teeth. Figure 3 shows the results of OMA for these two cases. Here, the OMA was done using the commercial package ARTeMIS.

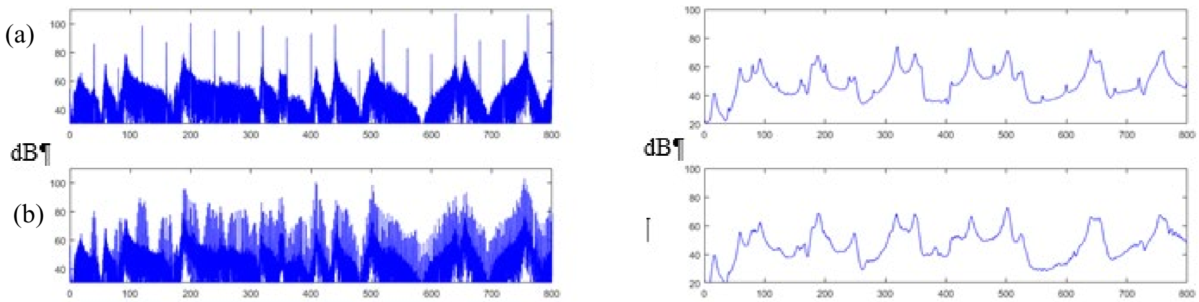

Figure 2: Simulated gearbox responses at (a) constant and (b) varying (±5%) speed.
(Left) Original spectra (Right) After exponential liftering

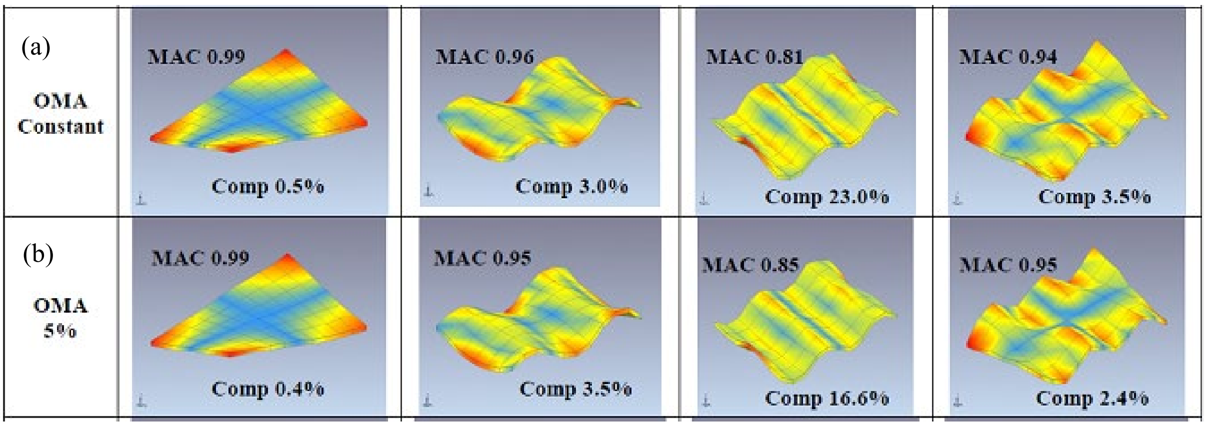

Figure 3: OMA results for simulated gearbox responses at constant and varying (±5%) speed.
(a) OMA mode shapes for constant speed (b) Ditto varying speed
MAC = modal assurance criterion; Comp = complexity.

A very important recent development in gear diagnostics was published in [5]. For a single stage gearset, running at different constant speeds or varying speed, it was found that after applying a comb lifter in the angle domain (i.e. after order tracking) to all rahmonics of the GM quefrency (period), which thus included all rahmonics of both gear rotational periods, the resulting log spectrum was periodic at the GM frequency, and thus resembled Mark's "DFT spectrum" [6]. The method was called the angle cepstrum comb liftering (ACCL) method, and thus obviously removed Mark's "mesh transfer functions", defined in the angle domain, to give something closely related to the elemental TEs of the gear pair. To achieve the equivalent of synchronous averaging, the processing had to be done over an integer number of hunting tooth (HT) periods, so that all rahmonics and harmonics were in discrete lines.

Figure 4 demonstrates that the protrusion of fault-related harmonics from the base noise level in the spectrum is the same for the same number of HT periods, independent of the speed, but for 10 Hz speed, the record length in seconds has to be twice as long as for 20 Hz.

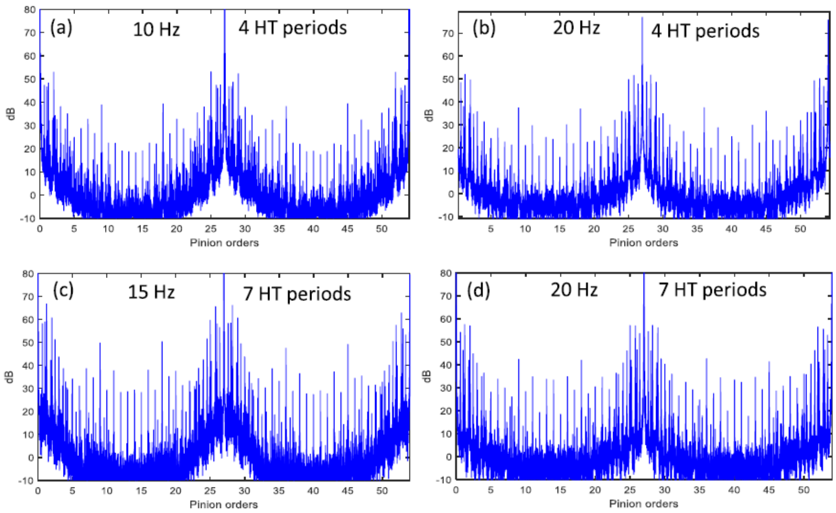

Figure 4: ACCL spectra for different speeds and number of HT periods

The ACCL spectra in [5] were for two types of faults with fairly uniform harmonics over a wide frequency range; tooth root cracks with a localised impulse response (IR), and distributed pitting with a broadband pseudo-random spectrum. It was demonstrated that by combining the relatively white ACCL amplitude spectrum (excluding the gearmesh harmonics and near sidebands) with the phase of the angle domain acceleration signal, information could be obtained on the localisation of the fault.

Figure 5 (from [5]) compares the resulting ACCL "time signal" (in the angle domain) with TE measurements from [7], made using shaft encoders, at different constant speeds, and separating GTE and (load dependent) compliance at the mesh. The STE is a weighted sum of the two, depending on the load. While somewhat noisier, it can be seen that the ACCL time signal is similar to the STE, and with the same time scale. The corresponding synchronously averaged acceleration signal had a somewhat longer local response because the resonances were not removed. For the case of distributed pitting, it was found that the ACCL ALR (average log

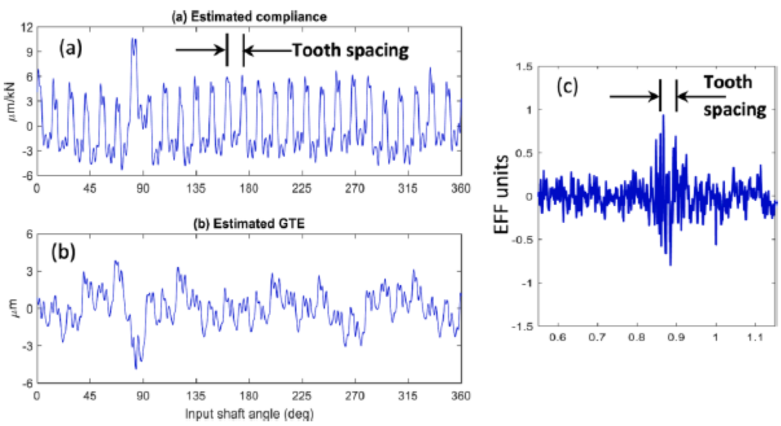

Figure 5: Measured STE components from [7] showing (a) compliance component (b) GTE component, and (c) Extract from ACCL record with same x-axis scaling

ratio, described in [8], but simplified) spectra correlated with both the percentage pitting area, and surface roughness over a 60-hour test.

A precursor to the ACCL method applied the inverse process on the same signals, including those at variable speed, to diagnose a bearing fault on the low speed shaft. This was achieved by comb notch liftering in the angle cepstrum to remove the gear signals, since the bearing fault signals were carried by broadband pseudo-cyclostationary (PCS) signals [9]. Even after removing the (additive) gear components, it was found that the envelope signals still had multiplicative (modulation) effects from the high speed gear (with the crack fault), but these could be greatly attenuated by processing the envelope signal, requiring further inverse and forward order tracking. The results were published in [10], but repeated in Chapter 7 of [11].

A final example is given as to how cepstral OMA can be used to remove the transfer functions (TFs) from gear response signals, using a blind source separation (BSS) method [12] to extract the response to a second order cyclostationary (CS2) source with a unique cyclic frequency. The TF was found from the broadband excitation (from tooth sliding), at a particular gearmesh (thus modulated by a unique cyclic frequency) after removal of shaft speed related components in the angle domain cepstrum. Those transfer functions could then be used to remove the TF effects in the shaft speed components from each gear, giving pseudo forcing function waveforms (effectively static TE), which should be the same independent of speed, and even for varying speed. Figure 6 compares the waveforms of the original acceleration responses at 10 Hz and 20 Hz running speed, with the pseudo forcing functions after TF removal. In this case the gears were healthy, and the static TE comes primarily from the changing stiffness at the toothmesh for switching between one and two tooth pairs in mesh. Even though the number of harmonics is restricted, it can be seen that the estimated forcing functions are approximately a square wave from the sudden change in stiffness. Even though this was done for a single stage gearbox, the different gearmesh frequencies in multiple stage gearboxes would be separable by virtue of their different cyclic frequencies.

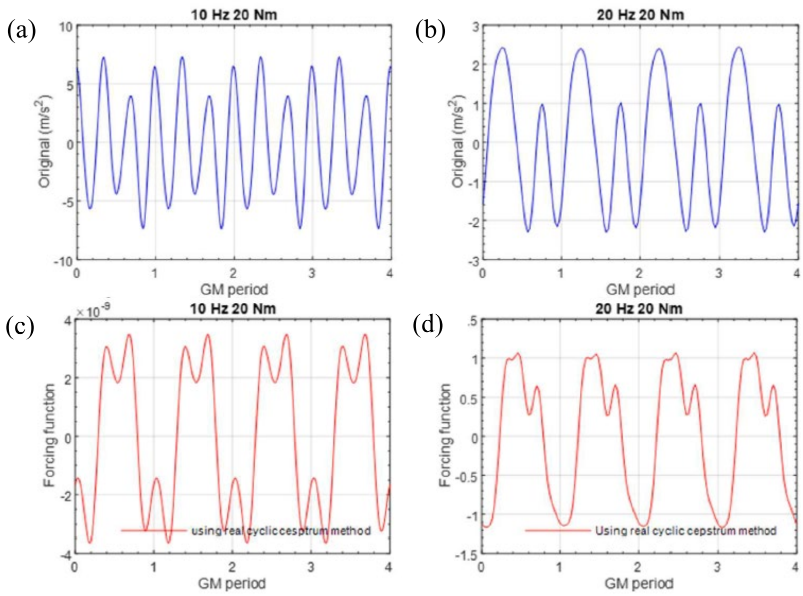

Figure 6: The original (response) signals (a, b) and the reconstructed forcing functions (c, d) obtained by the real cyclic cepstrum method for 10 Hz and 20 Hz.

Refs. [13, 14] give further examples of the application to gear TE and vibration for the case of a tooth root crack fault.

Conclusion

The paper demonstrates how the unique properties of the cepstrum give two types of separation that are very valuable in machine diagnostics; separation of different sources acting simultaneously, such as gears and bearings, and for each source, separation of the transfer functions to each measurement point from the common forcing function, the latter often being retrievable even in the case of operation at different constant speeds or varying speed along the record, where the response signals are very different.

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